What's the Science ?
Communication under Model uncertainty

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Scientific knowledge

- Scientific knowledge is a collection of representations of reality, **models**. Each of them offers different **explanations** to what we observe from the sensitive world.
- There is **no** such thing as a unique, permanent, **true** representation of reality.
  - → When the depository of Scientific authority speaks about scientific knowledge, he **cannot prove** what he claims.
  - → Communication over Science belongs to the realm of **non-certifiable** communication
- In that context, the **sender** is only assumed to have **more educated** perception of the existing models.

Model uncertainty

- Models can be seen as **stochastic** predictors of the **outcome** of given actions.
- Formally, they are **probability measures** over possible states of the world.
- When there is uncertainty over which one is the best one $\rightarrow$ **model uncertainty**.

Scientific communication

- I study a game of communication under model uncertainty, where there is an asymmetry of **interests**, and communication is **cost-less**.
  $\rightarrow$ Cheap-Talk

- The game is similar to the canonical one of [Crawford and Sobel, 1982] except that **messages** are over **models** (probability measures) and not **states of world**.

- As a result, receivers may be **ambiguity sensitive** regarding models. I will assume they hold smooth ambiguity preferences : KMM [Klibanoff et al., 2005].

Applications

There are many situation where there is an **asymmetry of interests** between the scientific authority and those who **act** in function of its recommendation :

- Company selling a **new technology** relying on a scientific theory (e.g. Long run effects of GMO)
- **Health authority** recommending a public behaviour (e.g. vaccination / contribution to a public good).
- IPCC predictions on **climate damages** to green house gas (GHG) emitters / contribution to a public bad.

Main results

1. All equilibrium are **partition equilibrium** : sender credibly points out a set of model.
2. When ambiguity aversion grows, it is **harder** (in terms of bias) to get a non-babbling equilibrium (saying something **credible**).
3. When receivers are MEU [Gilboa and Schmeidler, 1989] all equilibria can be **ranked** by informativness and the sender is better off playing **the most informative**.

Interpretation :

→ When it comes to models, assuming ambiguity aversion, it is **much harder to keep credibility** if there is a bias.

→ Yet, a credible sender can convey **much more information** than while talking about states (under MEU).

Related work

- Ambiguity in cheap talk over **states of the world** with ambiguity averse preferences has been introduced by [Kellner and Le Quement, 2017].This change is to the **advantage of the sender**.

- [Kellner and Le Quement, 2018] further allows to **Ellsbergian communication strategies**, strengthening this result.

- Cheap talk over **states of the world** with **multiple receivers** have been studied by [Goltsman and Pavlov, 2008]. To my knowledge, no work on **cheap talk prior** to a game of **contribution to a public good/bad**.

Timing

1. Nature selects the type of the sender, which is privately informed.
2. The sender sends a message to the receivers regarding its type.
3. The receivers chooses an action.

Receiver

- One receiver
- **Actions**: $s \in \mathcal{S} = [0,1]$ the action space of the receiver.
- $\Omega = \{H, L\}$
- $d : \mathcal{S} \times \Omega \to \mathbb{R}^+$, increasing and strictly convex in the first argument. $\forall s \in \mathcal{S}$; $d(s,L) \leq d(s,H)$
- **Pay-off functions** : $u(s) = s - d(s,\omega)$

Decision making

- Probability distributions are Bernoulli of parameter $\theta \in T = [\underline{\theta}, \overline{\theta}] \subset (0, 1)$ the set of types of the sender.
- $\mathcal{B}$ the set of all closed intervals of $[\underline{\theta}, \overline{\theta}]$, and $B \in \mathcal{B}$ the beliefs of the receiver.
- In situation of **ambiguity**, I assume the receiver to evaluate action s by:

$$V_B(s) = \int_{\theta \in [\underline{\theta}, \overline{\theta}]} \mu(\theta)\phi(s - \mathbb{E}_\theta(d(s, \omega)))d\theta$$

- $\mu \in \Delta([\underline{\theta}, \overline{\theta}])$ the second order **common prior** of both sender and receiver, $\phi : \mathbb{R} \to \mathbb{R}$ characterises attitude toward ambiguity.

Receiver's Equilibrium

When the set of beliefs of the receiver is B, he chooses s^* such that for all $s \in \mathcal{S}$:

$$V_B(s^*) \geq V_B(s)$$

$G : [\underline{\theta}, \overline{\theta}] \rightarrow T$ be the mapping that gives the **equilibrium action** of the receiver given his beliefs.

$$G(B) = argmax_{s \in \mathcal{S}} \int_{\theta \in B} \mu(\theta)\phi(s - \mathbb{E}_\theta(d(s), \omega))d\theta$$

$\rightarrow$ **concavity** implies that $G(B)$ always exists and is unique.

Sender

The utility of the sender given ω and action s is :

$$U_0(s,\omega) = s - d_0(s,\omega) \tag{1}$$

where $d_0 : \mathcal{S} \times \Omega \to \mathbb{R}^+$, increasing and strictly convex in the first argument. $\forall s \in \mathcal{S}$; $d_0(s,L) \leq d_0(s,H)$

- M the set of **messages** of the sender.
- A **strategy** for the sender is $\sigma : [\underline{\theta}, \overline{\theta}] \to M$ which consists in transmitting a message $m \in M$ to the receivers regarding its type
- Call Σ the set of the sender's strategy

Updating

Having received message m, receiver **updates** his prior using Bayes' rule such that :

$$\mu(\theta|m) = \frac{\mu(\theta)q(m|\theta)}{\int_{\theta\in[\underline{\theta},\bar{\theta}]} q(m|\theta)\mu(\theta)d\theta}$$

where $q(m|\theta))$ is the signalling rule for the sender. Call $B(m) = supp(\mu(\cdot|m))$, the updated belief of the receivers having received m. Receiver i then evaluates its strategies by

$$V_{B(m)}(s) = \int_{\theta\in\mathcal{C}} \mu(\theta|m)\phi(s - \mathbb{E}_\theta(d(s,\omega)))d\theta$$

Sender's Equilibrium

- $\sigma^{-1}(m) \in [\underline{\theta}, \overline{\theta}]$, for $m \in supp(\sigma)$, be the set of **potential types** of the sender, in the eyes of the receivers, having received message m
- Having learned its type θ_0, the sender evaluates strategy σ by:

$$V_{\theta_0}(\sigma(\theta_0)) = G(\sigma^{-1}(m)) - \mathbb{E}_{\theta_0}(d_0(G(\sigma^{-1}(m), \omega))$$

- At equilibrium, the sender chooses σ^* such that for all $\sigma \in \Sigma$:

$$V_{\theta_0}(\sigma^*(\theta_0)) \geq V_{\theta_0}(\sigma(\theta_0))$$

Bias

I further define $s_0(\theta_0) = argmax_{s \in \mathcal{S}} \mathbb{E}_{\theta_0}(U_0(s))$ the **optimal action** in the eyes of the sender. In the following I will assume that $\forall \theta \in T$:

$$G(\theta) < s_0(\theta) \text{ or } G(\theta) > s_0(\theta)$$

i.e. the sender and the receiver are always **biased** in the **same direction**.

Partition equilibrium

Definition

A partition equilibrium is a partition of the set of types : $\cup_k C_k = [\underline{\theta}, \overline{\theta}]$ such that the equilibrium strategy of the sender is $\sigma^*(C_k) = m_k$ and the receiver's action is $G(\theta(m_k))$

Proposition

Any equilibrium of the game is a partition equilibrium.

$\rightarrow$ Proof similar to [Crawford and Sobel, 1982]

Comparative statics

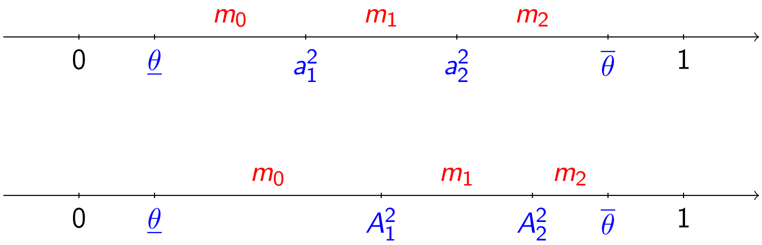
Proposition

Let $\underline{\theta} < a_1^q < ... < a_q^q < \overline{\theta}$ be the cutoff types of the equilibrium with q cutoffs for receivers with given ambiguity aversion and $\underline{\theta} < A_1^q < ... < A_q^q < \overline{\theta}$ be the cutoff types of the equilibrium with q cutoffs for the same receivers with increased ambiguity aversion. Then we have that for all $k \leq q$:

$$A_k^q > a_k^q$$

- In particular the **existence** of non-babbling equilibrium ($q = 1$) in the most ambiguity averse case.
  - $\rightarrow$ Harder (in terms of bias) to be credible under ambiguity aversion.
- Only **within** q cutoff types comparison.

Comparing equilibria



Non-babbling under MEU Equilibria

Assume ϕ is such that $-\frac{\phi''}{\phi'} \to +\infty$. Then the receiver behaves as if he had MEU preferences with belief $B(m)$.

Proposition

When $\forall\theta \in T$, $G(\theta) < s_0(\theta)$ the only equilibrium is the babbling equilibrium (type independent message, message independent action).

In the following I will assume that $\forall\theta \in T$, $G(\theta) > s_0(\theta)$

MEU Equilibria

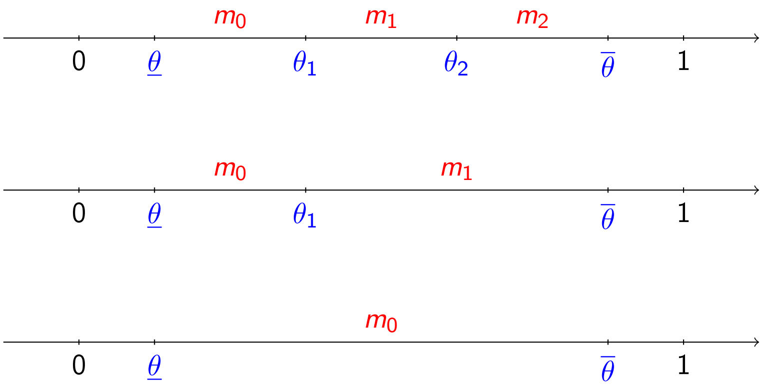
Proposition

There exists $\theta_1 < ... < \theta_N \in (\underline{\theta}, \overline{\theta})$ *such that the set of all equilibrium of the game is*
$\{(\sigma^*_q, G([\theta_k, \theta_{k+1}))) | \sigma^*_q([\theta_k, \theta_{k+1})) = m_k \text{ for } 0 \leq k \leq q\}_{0 \leq q \leq N}$

- There are several equilibrium characterised by their **number of cut-off types**.
- In all equilibrium, the the k-th cut-off type is the **same**.

Representation of equilibria

A direct consequence is that all equilibrium of the game can be **ranked** by informativeness, which will not be true for any ϕ.



Selection of equilibria

A direct corollary can be established regarding ex-ante equilibrium selection by the sender.

Corollary

1. *The sender is always ex-ante weakly better off by playing the most informative equilibrium strategy* $\forall q \geq 0$ *:* $V_{\theta_0}(\sigma_N^*(\theta_0)) \geq V_{\theta_0}(\sigma_q^*(\theta_0))$
2. *When the sender's type is not in* $[\theta_1, \theta_2)$*, it is ex-ante strictly better off playing the most informative equilibrium strategy.*

IPCC and Climate Agreements

- An interesting special case is when multiple receivers will play a game of **contribution to a public bad** (CPB) accordingly to the scientific announcement.
  → Communication over **climate damages** by the IPCC report before international climate agreements.
- Then, the overall level of GHG emitted will be **inefficient**.
- The sender could be seen as trying to act as a social planner, **internalising** all countries damages.
  → Sender is systematically **downwards biased** towards the total level of emission of the senders.

Receivers

- Two receivers
- **Actions**: $s_i \in \mathcal{S} = [0, 1]$ the normalised level of emission of receiver i. $E = [0, 2]$ total emissions.
- $u_1(s_1, s_2) = -(s_1 + s_2 - \omega)^2$
- $u_2(s_2, s_1) = -(s_1 + s_2 - \omega - b)^2$, with $b > 0$
- An equilibrium in the receivers game is defined similarly to a Nash equilibrium using the receivers value function V_B^i.

Sender

The sender cares only of **maximising total welfare**. The utility of the sender given ω and total emission e is :

$$U_0(e) = -(e-\omega)^2 - (e-\omega-b)^2$$

Having learned its type θ_0, the sender evaluates strategy σ by:

$$V_{\theta_0}(\sigma(\theta_0)) = \mathbb{E}_{\theta_0}(U_0(\sigma^{-1}(m)))$$

At equilibrium, the sender chooses σ^* such that for all $\sigma \in \Sigma$:

$$V_{\theta_0}(\sigma^*(\theta_0)) \geq V_{\theta_0}(\sigma(\theta_0))$$

Credible communication

Assume receivers evaluate strategies by $min_{\theta \in B} \mathbb{E}_\theta(u_i(s_i, s_{-i}))$. A non-babbling equilibrium **exists** if and only if :

$$b \leq \frac{2(1-\underline{\theta})}{3}$$

Yet, when receivers are ambiguity neutral and have a uniform prior, a non-babbling equilibrium **exists** if and only if :

$$b \leq 3$$

- In a game of contribution to a public bad, the sender is credible if and only if contributor's valuation is **close enough.**
- The non-babbling equilibrium is **less likely to exist** when receivers are MEU.